SADLER. UNIT 4. CHAPTER 9

EXERCISE 9A

Q1. $\int 60x(x^2-3)^5\,dx$

Let $u = x^2-3$

$\frac{du}{dx} = 2x$

$dx = \frac{1}{2x}\,du$

$= \int \frac{60x}{2x} u^5\,du$

$= \int 30u^5\,du$

$= \frac{30u^6}{6} + c$

$= 5u^6 + c$

$= 5(x^2-3)^6 + c$

Q2. $\int 80x(1-2x)^3\,dx$

Let $u = 1-2x$

$\frac{du}{dx} = -2$

$dx = -\frac{1}{2}\,du$

$= \int 80x\,u^3\left(-\frac{1}{2}\right)du$

$= \int -40x\,u^3\,du$

$2x = 1-u$

$x = \frac{1-u}{2}$

$= \int -20(1-u)u^3\,du$

$= \int -20u^3 + 20u^4\,du$

$= \frac{-20u^4}{4} + \frac{20u^5}{5} + c$

$= -5(1-2x)^4 + 4(1-2x)^5 + c$

$= -(1-2x)^4[5-4(1-2x)] + c$

$= -(1-2x)^4(1+8x) + c$

Q3. $\int 12x(3x+1)^5\,dx$

Let $u = 3x+1$

$\frac{du}{dx} = 3$

$dx = \frac{1}{3}\,du$

$= \int 12x\,u^5\left(\frac{1}{3}\right)du$

$= \int 4x\,u^5\,du$

$u-1 = 3x$

$x = \frac{u-1}{3}$

$= \int \frac{4}{3}(u-1)u^5\,du$

$= \int \frac{4}{3}u^6 - \frac{4}{3}u^5\,du$

$= \frac{4u^7}{21} - \frac{4u^6}{18} + c$

$= \frac{4(3x+1)^7}{21} - \frac{4(3x+1)^6}{18} + c$

$= \frac{4(3x+1)^6}{3}\left(\frac{1}{7}(3x+1) - \frac{1}{6}\right) + c$

$= \frac{4}{3}(3x+1)^6\left(\frac{6(3x+1)-7}{42}\right) + c$

$= \frac{2}{63}(3x+1)^6(18x-1) + c$

Q4. $\int 6x(2x^2-1)^5\,dx$

Let $u = 2x^2-1$

$\frac{du}{dx} = 4x$

$dx = \frac{1}{4x}\,du$

$= \int \frac{6x}{4x}u^5\,du$

$= \int \frac{3}{2}u^5\,du$

$= \frac{3u^6}{12} + c$

$= \frac{u^6}{4} + c$

$= \frac{(2x^2-1)^6}{4} + c$

Q5 $\int 12x(3x^2+1)^5\,dx$

Let $u = 3x^2+1$

$\frac{du}{dx} = 6x$

$dx = \frac{1}{6x}\,du$

$= \int \frac{12x}{6x} u^5\,du$

$= \int 2u^5\,du$

$= \frac{2u^6}{6} + c$

$= \frac{1}{3}(3x^2+1)^6 + c$

Q6. $\int 3x(x-2)^5\,dx$

Let $u = x-2$

$\frac{du}{dx} = 1$

$dx = du$

$= \int 3x u^5\,du$

$x = u+2$

$= \int 3(u+2)u^5\,du$

$= \int 3u^6 + 6u^5\,du$

$= \frac{3u^7}{7} + \frac{6u^6}{6} + c$

$= \frac{3}{7}(x-2)^7 + (x-2)^6 + c$

$= (x-2)^6\left(\frac{3}{7}(x-2)+1\right) + c$

$= \frac{1}{7}(x-2)^6(3x-6+7) + c$

$= \frac{1}{7}(x-2)^6(3x+1) + c$

Q7. $\int 20x(3-x)^3\,dx$

Let $u = 3-x \Rightarrow x = 3-u$

$\frac{du}{dx} = -1$

$du = -dx$

$= \int 20x\,u^3 - du.$

$= \int -20(3-u)u^3\,du$

$= \int -60u^3 + 20u^4\,du$

$= \frac{-60u^4}{4} + \frac{20u^5}{5} + c$

$= -15(3-x)^4 + 4(3-x)^5 + c$

$= -(3-x)^4(15-4(3-x)) + c$

$= -(3-x)^4(15-12+4x) + c$

$= -(3-x)^4(3+4x) + c$

Q8. $\int 4x(5-2x)^5\,dx$

Let $u = 5-2x$

$\frac{du}{dx} = -2$

$dx = -\frac{1}{2}\,du$

$= \int 4x u^5 \left(-\frac{1}{2}\right) du$

$= \int -2x u^5\,du$

$u - 5 = -2x$

$= \int (u-5)u^5\,du$

$= \int u^6 - 5u^5\,du$

$= \frac{u^7}{7} - \frac{5u^6}{6} + c$

$= \frac{(5-2x)^7}{7} - \frac{5(5-2x)^6}{6} + c$

$= (5-2x)^6\left(\frac{1}{7}(5-2x) - \frac{5}{6}\right) + c$

$= (5-2x)^6\left(\frac{5}{7} - \frac{2}{7}x - \frac{5}{6}\right) + c.$

$= (5-2x)^6\left(\frac{30-12x-35}{42}\right) + c$

$= (5-2x)^6\left(\frac{-5-12x}{42}\right) + c$

$= -\frac{1}{42}(5-2x)^6(5+12x) + c$

Q9. $\int 20x(2x+3)^3\,dx$

Let $u = 2x+3$

$\frac{du}{dx} = 2$

$dx = \frac{1}{2}\,du$

$= \int 10x\,u^3\,du$

$\frac{u-3}{2} = x$

$= \int 5(u-3)u^3\,du$

$= \int 5u^4 - 15u^3\,du$

$= \frac{5u^5}{5} - \frac{15u^4}{4} + c$

$= u^5 - \frac{15}{4}u^4 + c$

$= (2x+3)^5 - \frac{15}{4}(2x+3)^4 + c$

$= (2x+3)^4\left(2x+3 - \frac{15}{4}\right) + c$

$= (2x+3)^4\left(\frac{8x+12-15}{4}\right) + c$

$= \frac{1}{4}(2x+3)^4(8x-3) + c$

Q10. $\int 18x\sqrt{3x+1}\,dx$

Let $u = 3x+1$

$\frac{du}{dx} = 3$

$dx = \frac{1}{3}\,du$

$= \int 6x\sqrt{u}\,du$

$\frac{u-1}{3} = x$

$= \int 2(u-1)\sqrt{u}\,du$

$= \int 2\sqrt{u^3} - 2\sqrt{u}\,du$

$= \int 2u^{\frac{3}{2}} - 2u^{\frac{1}{2}}\,du$

$= \frac{2u^{\frac{5}{2}}}{\left(\frac{5}{2}\right)} - \frac{2u^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + c$

$= \frac{4u^{\frac{5}{2}}}{5} - \frac{4u^{\frac{3}{2}}}{3} + c$

$= \frac{12u^{\frac{5}{2}} - 20u^{\frac{3}{2}} + c}{15}$

$= \frac{1}{15}u^{\frac{3}{2}}(12u - 20) + c$

$= \frac{1}{15}\sqrt{(3x+1)^3}\,(36x+12-20) + c$

$= \frac{4}{15}\sqrt{(3x+1)^3}\,(9x-2) + c$

Q11. $\int \frac{6x}{\sqrt{3x^2+5}}\,dx$

Let $u = 3x^2+5$

$\frac{du}{dx} = 6x$

$dx = \frac{1}{6x}\,du$

$= \int \frac{1}{\sqrt{u}}\,du$

$= \int u^{-\frac{1}{2}}\,du$

$= 2u^{\frac{1}{2}} + c$

$= 2\sqrt{3x^2+5} + c$

Q12. $\int \frac{3x}{\sqrt{1-2x}}\,dx$

Let $u = 1-2x$

$\frac{du}{dx} = -2$

$dx = -\frac{1}{2}\,du$

$= \int \frac{3x}{\sqrt{u}}\left(-\frac{1}{2}\right)du$

$u - 1 = -2x$

$x = \frac{1-u}{2}$

$= \int -\frac{3(1-u)}{4\sqrt{u}}\,du.$

$= \int \frac{3u}{4\sqrt{u}} - \frac{3}{4\sqrt{u}}\,du$

$= \int \frac{3u^{\frac{1}{2}}}{4} - \frac{3}{4}u^{-\frac{1}{2}}\,du$

$= \frac{\frac{3}{4}u^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} - \frac{\frac{3}{4}u^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} + c$

$= \frac{1}{2}u^{\frac{3}{2}} - \frac{3}{2}u^{\frac{1}{2}} + c$

$= \frac{1}{2}\sqrt{u}\,(u-3) + c$

$= -\sqrt{1-2x}\,(x+1) + c$

Q13. $\int 8\sin^5(2x)\cos(2x)\,dx$

Let $u = \sin(2x)$

$\frac{du}{dx} = 2\cos(2x)$

$du = 2\cos(2x)\,dx$

$= \int 4u^5\,du$

$= \frac{4u^6}{6} + c$

$= \frac{2\sin^6(2x)}{3} + c$

Q14. $\int 27\cos^7(3x)\sin(3x)\,dx$

Let $u = \cos(3x)$

$\frac{du}{dx} = -3\sin(3x)$

$du = -3\sin(3x)\,dx$

$= \int -9u^7\,du$

$= -\frac{9u^8}{8} + c$

$= -\frac{9\cos^8(3x)}{8} + c$

Q15. $\int 6x\sin(x^2+4)\,dx$

Let $u = x^2+4$

$\frac{du}{dx} = 2x$

$dx = \frac{1}{2x}\,du$

$= \int 3\sin u\,du.$

$= -3\cos u + c$

$= -3\cos(x^2+4) + c$

Q16. $\int (4x+3)(2x+1)^5\,dx$

Let $u = 2x+1$

$\frac{du}{dx} = 2$

$dx = \frac{1}{2}\,du$

$= \int \frac{1}{2}(4x+3)u^5\,du$

$\frac{u-1}{2} = x$

$= \int \frac{1}{2}\left(2(u-1)+3\right)u^5\,du$

$= \int (u-1+\frac{3}{2})u^5\,du$

$= \int u^6 + \frac{1}{2}u^5\,du$

$= \frac{u^7}{7} + \frac{u^6}{12} + c$

$= \frac{12u^7 + 7u^6}{84} + c$

$= \frac{1}{84}u^6(12u+7) + c$

$= \frac{1}{84}(2x+1)^6(24x+12+7) + c$

$= \frac{1}{84}(2x+1)^6(24x+19) + c$

EXERCISE 9B

Q1. $\int x + \sin 3x \, dx$

$= \frac{x^2}{2} - \frac{1}{3}\cos 3x + c$

Q2. $\int 2 \, dx$

$= 2x + c$

Q3. $\int \sin 8x \, dx$

$= -\frac{1}{8}\cos 8x + c$

Q4. $\int (\cos x + \sin x)(\cos x - \sin x) \, dx$

$= \int \cos^2 x - \sin^2 x \, dx$

$= \int \cos 2x \, dx$

$= \frac{1}{2}\sin(2x) + c$

Q5. $\int \frac{x^2 + x}{\sqrt{x}} \, dx$

$= \int x^{\frac{3}{2}} + x^{\frac{1}{2}} \, dx$

$= \frac{2x^{\frac{5}{2}}}{5} + \frac{2x^{\frac{3}{2}}}{3} + c$

$= \frac{2\sqrt{x^5}}{5} + \frac{2\sqrt{x^3}}{3} + c$

Q6. $\int 4x \sin x^2 \, dx$

Let $u = x^2$

$\frac{du}{dx} = 2x$

$dx = \frac{1}{2x} du$

$= \int 2 \sin u \, du$

$= -2\cos u + c$

$= -2\cos(x^2) + c$

Q7. $\int 8x \sin(x^2 - 3) \, dx$

Let $u = x^2 - 3$

$\frac{du}{dx} = 2x$

$dx = \frac{1}{2x} du$

$= \int 4 \sin u \, du$

$= -4\cos u + c$

$= -4\cos(x^2 - 3) + c$

Q8. $\int 24\sqrt{1+3x} \, dx$

Let $u = 1 + 3x$

$\frac{du}{dx} = 3$

$dx = \frac{1}{3} du$

$= \int 8\sqrt{u} \, du$

$= \frac{8u^{\frac{3}{2}}}{\frac{1}{2}} + c$

$= 16u^{\frac{3}{2}} + c$

$= 16(1+3x)^{\frac{3}{2}} + c$

Q9. $\int 15x\sqrt{1+3x} \, dx$

Let $u = 1 + 3x$

$\frac{du}{dx} = 3$

$dx = \frac{1}{3} du$

$= \int 15x\sqrt{u} \left(\frac{1}{3}\right) du$

$= \int 5x\sqrt{u} \, du$

$\frac{u-1}{3} = x$

$= \int \frac{5\sqrt{u}}{3}(u-1) \, du$

$= \int \frac{5}{3}u^{\frac{3}{2}} - \frac{5}{3}u^{\frac{1}{2}} \, du$

$= \frac{\frac{5}{3}u^{\frac{5}{2}}}{\left(\frac{5}{2}\right)} - \frac{\frac{5}{3}u^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + c$

$= \frac{2}{3}u^{\frac{5}{2}} - \frac{10}{9}u^{\frac{3}{2}} + c$

$= \frac{6}{9}u^{\frac{5}{2}} - \frac{10}{9}u^{\frac{3}{2}} + c$

$= \frac{2}{9}u^{\frac{3}{2}}(3u-5)+c$

$= \frac{2}{9}(1+3x)^{\frac{3}{2}}(3+9x-5)+c$

$= \frac{2}{9}\sqrt{(1+3x)^3}(9x-2)+c$

Q10 $\int \sin^4(2x)\cos(2x)\,dx$

Let $u=\sin(2x)$

$\frac{du}{dx} = 2\cos(2x)$

$du = 2\cos(2x)\,dx$

$\frac{1}{2}du = \cos(2x)\,dx$

$= \int \frac{1}{2}u^4\,du$

$= \frac{1}{10}u^5 + c$

$= \frac{1}{10}\sin^5(2x)+c$

Q11. $\int 6x(2x+7)^5\,dx$

Let $u = 2x+7$

$\frac{du}{dx} = 2$

$dx = \frac{1}{2}du$

$u-7 = 2x$

$6x = 3u-21$

$= \int (3u-21)u^5(\frac{1}{2})\,du$

$= \int \frac{3}{2}u^6 - \frac{21}{2}u^5\,du$

$= \frac{3u^7}{14} - \frac{21u^6}{12} + c$

$= \frac{3}{2}u^6(\frac{1}{7}u - \frac{7}{6}) + c$

$= \frac{3}{2}u^6(\frac{6u-49}{42}) + c$

$= \frac{3}{84}u^6(6u-49)+c$

$= \frac{3}{84}(2x+7)^6(12x+42-49)+c$

$= \frac{3}{84}(2x+7)^6(12x-7)+c$

$= \frac{1}{28}(2x+7)^6(12x-7)+c$

Q12 $\int 6(2x+7)^5\,dx$

Let $u=2x+7$

$\frac{du}{dx} = 2$

$dx = \frac{1}{2}du$

$= \int 3u^5\,du$

$= \frac{3u^6}{6}+c$

$= \frac{1}{2}u^6 + c$

$= \frac{1}{2}(2x+7)^6+c$

Q13. $\int 3x^2-2\,dx$

$= \frac{3x^3}{3} - 2x + c$

$= x^3 - 2x + c$

Q14 $\int 4x(3x^2-2)^7\,dx$

Let $u = 3x^2-2$

$\frac{du}{dx} = 6x$

$dx = \frac{1}{6x}du$

$= \int \frac{4x}{6x}u^7\,du$

$= \int \frac{2}{3}u^7\,du$

$= \frac{2u^8}{24} + c$

$= \frac{1}{12}u^8 + c$

$= \frac{1}{12}(3x^2-2)^8 + c$

Q15 $\int \cos x + \sin 2x\,dx$

$= \sin x - \frac{1}{2}\cos(2x) + c$

Q16 $\int 6x(3x-2)^7\,dx$

Let $u = 3x-2$

$\frac{du}{dx} = 3$

$dx = \frac{1}{3}du$

$= \int 2x\, u^7\, du$

$\dfrac{u+2}{3} = x$

$= \int \frac{2}{3}(u+2)\, u^7\, du$

$= \frac{2}{3}\int u^8 + 2u^7\, du$

$= \frac{2}{3}\left(\frac{u^9}{9} + \frac{2u^8}{8}\right) + c$

$= \frac{2}{3}\left(\frac{4u^9 + 9u^8}{36}\right) + c$

$= \frac{1}{54}\, u^8(4u+9) + c$

$= \frac{1}{54}(3x-2)^8(12x-8+9) + c$

$= \frac{1}{54}(3x-2)^8(12x+1) + c$

Q17. $\int x\, dx$

$= \dfrac{x^2}{2} + c$

Q18 $\displaystyle\int \frac{6}{\sqrt{1+2x}}\, dx$

Let $u = 1+2x$

$\dfrac{du}{dx} = 2$

$dx = \frac{1}{2}\, du$

$= \int \frac{3}{\sqrt{u}}\, du$

$= \int 3u^{-\frac{1}{2}}\, du$

$= \dfrac{3u^{\frac{1}{2}}}{\frac{1}{2}} + c$

$= 6\sqrt{u} + c$

$= 6\sqrt{1+2x} + c$

Q19 $\displaystyle\int \frac{6x}{\sqrt{1+2x}}\, dx$

Let $u = 1+2x$

$\dfrac{du}{dx} = 2$

$dx = \frac{1}{2}\, du$

$= \int \frac{3x}{\sqrt{u}}\, du$

$\dfrac{u-1}{2} = x$

$= \int \frac{3}{2}(u-1)\, u^{-\frac{1}{2}}\, du$

$= \frac{3}{2}\int \left(u^{\frac{1}{2}} - u^{-\frac{1}{2}}\right) du$

$= \frac{3}{2}\left(\dfrac{u^{\frac{3}{2}}}{\frac{3}{2}} - \dfrac{u^{\frac{1}{2}}}{\frac{1}{2}}\right) + c$

$= u^{\frac{3}{2}} - 3u^{\frac{1}{2}} + c$

$= u^{\frac{1}{2}}(u-3) + c$

$= \sqrt{1+2x}\,(1+2x-3) + c$

$= \sqrt{1+2x}\,(2x-2) + c$

$= 2\sqrt{1+2x}\,(x-1) + c$

Q20. $\int (x^2+x+1)^8 (2x+1)\, dx$

Let $u = x^2+x+1$

$\dfrac{du}{dx} = 2x+1$

$du = (2x+1)\, dx.$

$= \int u^8\, du$

$= \dfrac{u^9}{9} + c$

$= \frac{1}{9}(x^2+x+1) + c$

Q21. $\int 24x \sin(x^2+3)\, dx$

let $u = x^2+3$

$\dfrac{du}{dx} = 2x$

$dx = \frac{1}{2x}\, du$

$= \int 12 \sin u\, du$

$= -12\cos u + c$

$= -12\cos(x^2+3) + c$

Q22. $\int (2x+1)\sqrt[3]{x-5}\,dx$

Let $u = x-5$

$\frac{du}{dx} = 1$

$du = dx$

$x = u+5$

$= \int (2(u+5)+1)\sqrt[3]{u}\,du$

$= \int (2u+10+1)u^{\frac{1}{3}}\,du$

$= \int 2u^{\frac{4}{3}} + 11u^{\frac{1}{3}}\,du$

$= \frac{2u^{\frac{7}{3}}}{\left(\frac{7}{3}\right)} + \frac{11u^{\frac{4}{3}}}{\left(\frac{4}{3}\right)} + c$

$= \frac{6}{7}u^{\frac{7}{3}} + \frac{33}{4}u^{\frac{4}{3}} + c$

$= \frac{24}{28}u^{\frac{7}{3}} + \frac{231}{28}u^{\frac{4}{3}} + c$

$= \frac{3}{28}u^{\frac{4}{3}}(8u+77) + c$

$= \frac{3}{28}(x-5)^{\frac{4}{3}}(8x-40+77) + c$

$= \frac{3}{28}\sqrt[3]{(x-5)^4}(8x+37) + c$

Q23. $\int \frac{(\sqrt{x}+5)^5}{\sqrt{x}}\,dx$

Let $u = \sqrt{x}+5$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$2\,du = \frac{1}{\sqrt{x}}\,dx$

$= \int 2u^5\,du$

$= \frac{2u^6}{6} + c$

$= \frac{1}{3}(\sqrt{x}+5)^6 + c$

Q24. $\int 4(2x-1)^5\,dx$

Let $u = 2x-1$

$\frac{du}{dx} = 2$

$dx = \frac{1}{2}\,du$

$= \int 2u^5\,du$

$= \frac{2u^6}{6} + c$

$= \frac{1}{3}(2x-1)^6 + c$

Q25. $\int 4x(2x-1)^5\,dx$

Let $u = 2x-1$

$\frac{du}{dx} = 2$

$dx = \frac{1}{2}\,du$

$= \int 4x\,u^5\left(\frac{1}{2}\right)du$

$= \int 2x\,u^5\,du$

$2x = u+1$

$= \int (u+1)u^5\,du$

$= \int u^6 + u^5\,du$

$= \frac{u^7}{7} + \frac{u^6}{6} + c$

$= \frac{1}{42}u^6(6u+7) + c$

$= \frac{1}{42}(2x-1)^6(12x-6+7) + c$

$= \frac{1}{42}(2x-1)^6(12x+1) + c$

Q26. $\int \cos^3(6x)\sin(6x)\,dx$

Let $u = \cos 6x$

$\frac{du}{dx} = -6\sin 6x$

$-\frac{1}{6}\,du = \sin 6x\,dx$

$= \int -\frac{1}{6}u^3\,du$

$= -\frac{u^4}{24} + c$

$= -\frac{1}{24}\cos^4(6x) + c$

Q27. $\int \frac{6x}{\sqrt{x^2-3}}\,dx$

Let $u = x^2 - 3$

$\frac{du}{dx} = 2x$

$dx = \frac{1}{2x}\,du$

$= \int \frac{3}{\sqrt{u}}\,du$

$= \int 3u^{-\frac{1}{2}}\,du$

$= \frac{3u^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} + c$

$= 6u^{\frac{1}{2}} + c$

$= 6\sqrt{x^2-3} + c$

Q28. $\int \sin 2x \cos 2x\,dx$

$= \int \frac{1}{2}\sin 4x\,dx$

$= -\frac{1}{8}\cos 4x + c$

Q29. $\int 8x^2(2x-1)^5\,dx$

Let $u = 2x - 1$

$\frac{du}{dx} = 2$

$dx = \frac{1}{2}\,du$

$= \int 4x^2 u^5\,du$

$u + 1 = 2x$

$4x^2 = (u+1)^2$

$= \int (u+1)^2 u^5\,du$

$= \int (u^2 + 2u + 1)u^5\,du$

$= \int u^7 + 2u^6 + u^5\,du$

$= \frac{u^8}{8} + \frac{2u^7}{7} + \frac{u^6}{6} + c$

$= u^6\left(\frac{u^2}{8} + \frac{2u}{7} + \frac{1}{6}\right) + c$

HCF$(6,7,8) = \frac{6\times 7\times 8}{2}$

$= 168$

$= u^6\left(\frac{21u^2 + 48u + 28}{168}\right) + c$

$= \frac{1}{168}u^6(21u^2 + 48u + 28) + c$

$= \frac{1}{168}(2x-1)^6\left(21(4x^2-4x+1) + 48(2x-1) + 28\right) + c$

$= \frac{1}{168}(2x-1)^6(84x^2 - 84x + 21 + 96x - 48 + 28) + c$

$= \frac{1}{168}(2x-1)^6(84x^2 + 12x + 1) + c$

EXERCISE 9C

Q1. $\int_0^1 16(2x+1)^3\,dx$

Let $u = 2x + 1$

$\frac{du}{dx} = 2$

$dx = \frac{1}{2}\,du$

when $x = 0$, $u = 1$

$x = 1$, $u = 3$.

$= \int_1^3 16u^3\left(\frac{1}{2}\right)du$

$= \int_1^3 8u^3\,du$

$= \left[2u^4\right]_1^3$

$= 2(81) - 2$

$= 160$

Q2. $\int_0^1 16x(2x+1)^3\,dx$

Let $u = 2x + 1 \Rightarrow 2x = u - 1$

$\frac{du}{dx} = 2$

$dx = \frac{1}{2}\,du$

when $x = 0$, $u = 1$

$x = 1$, $u = 3$.

$$= \int_1^3 16x\,u^3 \left(\tfrac{1}{2}\right) du$$

$$= \int_1^3 8x u^3\, dx$$

$$= \int_1^3 4(u-1)u^3\, du$$

$$= \int_1^3 4(u^4 - u^3)\, du$$

$$= 4\left[\frac{u^5}{5} - \frac{u^4}{4}\right]_1^3$$

$$= 4\left[\frac{243}{5} - \frac{81}{4} - \left(\frac{1}{5} - \frac{1}{4}\right)\right]$$

$$= 4\left[\frac{972 - 405 - 4 + 5}{20}\right]$$

$$= \frac{568}{5}$$

$$= \underline{\underline{113.6}}$$

Q3. $\int_0^1 \frac{6x}{25}(x+5)^4\, dx$

Let $u = x+5$

$\frac{du}{dx} = 1$

$dx = du$

When $x = 0$, $u = 5$

$x = 1$, $u = 6$

$$= \int_5^6 \frac{6(u-5)}{25} u^4\, du$$

$$= \frac{6}{25}\int_5^6 u^5 - 5u^4\, du$$

$$= \frac{6}{25}\left[\frac{u^6}{6} - \frac{5u^5}{5}\right]_5^6$$

$$= \frac{6}{25}\left[6^5 - 6^5 - \frac{5^6}{6} + 5^5\right]$$

$$= \frac{6}{25}\left[\frac{6\cdot 5^5 - 5^6}{6}\right]$$

$$= \frac{5^5(6-5)}{25}$$

$$= \underline{\underline{125}}$$

Q4. $\int_0^{\frac{\pi}{2}} 12\sin^5 x \cos x\, dx$

Let $u = \sin x$

$\frac{du}{dx} = \cos x$

$du = \cos x\, dx$

When $x = 0$, $u = 0$

$x = \frac{\pi}{2}$, $u = 1$

$$= \int_0^1 12u^5\, du.$$

$$= \left[\frac{12u^6}{6}\right]_0^1$$

$$= \left[2u^6\right]_0^1$$

$$= \underline{\underline{2}}$$

Q5. $\int_2^6 \frac{3x}{\sqrt{5x+6}}\, dx$

Let $u = 5x+6$

$\frac{du}{dx} = 5$

$dx = \frac{1}{5}du.$

when $x = 2$, $u = 16$

$x = 6$, $u = 36$

$$= \int_{16}^{36} \frac{3x}{\sqrt{u}}\left(\frac{1}{5}\right) du.$$

$$\frac{u-6}{5} = x$$

$$= \frac{1}{5}\int_{16}^{36} \frac{3(u-6)}{5} u^{-\frac{1}{2}}\, du$$

$$= \frac{3}{25}\int_{16}^{36} u^{\frac{1}{2}} - 6u^{-\frac{1}{2}}\, du$$

$$= \frac{3}{25}\left[\frac{2u^{\frac{3}{2}}}{3} - 12u^{\frac{1}{2}}\right]_{16}^{36}$$

$$= \frac{3}{25}\left[\frac{2}{3}(36)^{\frac{3}{2}} - 12(36)^{\frac{1}{2}} - \frac{2}{3}(16)^{\frac{3}{2}} + 12(16)^{\frac{1}{2}}\right]$$

$$= \frac{3}{25}\left[\frac{2}{3}(216) - 72 - \frac{2}{3}(64) + 48\right]$$

$$= \frac{3}{25}\left[144 - 72 + 48 - \frac{128}{3}\right]$$

$$= \frac{3}{25}\left(\frac{232}{3}\right) = \frac{232}{25} = 9.28\,//.$$

Q6. $\int_2^5 \frac{x+3}{\sqrt{x-1}}\, dx$

Let $u = x-1 \Rightarrow u+1 = x$

$\frac{du}{dx} = 1$

$dx = du$

When $x=2$, $u=1$

$x=5$, $u=4$

$= \int_1^4 \frac{u+4}{\sqrt{u}}\, du$

$= \int_1^4 u^{\frac{1}{2}} + 4u^{-\frac{1}{2}}\, du$

$= \left[\frac{2u^{\frac{3}{2}}}{3} + \frac{4u^{\frac{1}{2}}}{(\frac{1}{2})} \right]_1^4$

$= \left[\frac{2}{3}u^{\frac{3}{2}} + 8u^{\frac{1}{2}} \right]_1^4$

$= \left[\frac{2}{3}(4)^{\frac{3}{2}} + 8(2) - \frac{2}{3} - 8 \right]$

$= \frac{2}{3}(8) + 16 - \frac{2}{3} - 8$

$= \frac{14}{3} + 8$

$= \frac{38}{3}$

Q7. $\int_0^1 \frac{4}{\sqrt{2x+1}}\, dx$

Let $u = 2x+1$

$\frac{du}{dx} = 2$

$dx = \frac{1}{2} du$

When $x=0$, $u=1$

$x=4$, $u=9$

$= \int_1^9 \frac{2}{\sqrt{u}}\, du$

$= \int_1^9 2u^{-\frac{1}{2}}\, du$

$= \left[4u^{\frac{1}{2}} \right]_1$

$= 12 - 4$

$= 8$ units^2 //

Q8. $\int_0^3 6x(x-3)^3\, dx$

Let $u = x-3$

$\frac{du}{dx} = 1$

$dx = du$

When $x=0$, $u=-3$

$x=3$, $u=0$

$= \int_{-3}^0 6x\, u^3\, du$

$= \int_{-3}^0 6(u+3)\, u^3\, du$

$= \int_{-3}^0 6u^4 + 18u^3\, du$

$= 6\int_{-3}^0 u^4 + 3u^3\, du$

$= 6\left[\frac{u^5}{5} + \frac{3u^4}{4} \right]_{-3}^0$

$= 6\left[0 - \left(\frac{-243}{5} + \frac{243}{4} \right) \right]$

$= 6\left[\frac{243}{5} - \frac{243}{4} \right]$

$= 6\left(\frac{4(243) - 5(243)}{20} \right)$

$= \frac{-3(243)}{10}$

$A = \left| -\frac{729}{10} \right|$

$= 72.9$ units^2

EXERCISE 9D

Q1. $\int \cos 5x \cos 4x \, dx$

$= \int \frac{1}{2} [\cos 9x + \cos x] \, dx$

$= \frac{1}{2} \int \cos 9x + \cos x \, dx$

$= \frac{1}{2} [\frac{1}{9} \sin 9x + \sin x] + c$

$= \frac{1}{18} \sin 9x + \frac{1}{2} \sin x + c$

Q2. $\int \sin 7x \sin x \, dx$

$= \int \frac{1}{2} [\cos 6x - \cos 8x] \, dx$

$= \frac{1}{2} \int \cos 6x - \cos 8x \, dx$

$= \frac{1}{2} [\frac{1}{6} \sin 6x - \frac{1}{8} \sin 8x] + c$

$= \frac{1}{12} \sin 6x - \frac{1}{16} \sin 8x + c$

Q3. $\int \sin^4 x \cos x \, dx$

Let $u = \sin x$

$\frac{du}{dx} = \cos x$

$du = \cos x \, dx$

$= \int u^4 \, du$

$= \frac{u^5}{5} + c$

$= \frac{\sin^5 x}{5} + c$

Q4. $\int 6 \sin^3 x \cos x \, dx$

Let $u = \sin x$

$\frac{du}{dx} = \cos x$

$du = \cos x \, dx$

$= \int 6u^3 \, du$

$= \frac{6u^4}{4} + c$

$= \frac{3 \sin^4 x}{2} + c$

Q5. $\int \sin^3 x \, dx$

$= \int (1 - \cos^2 x) \sin x \, dx$

$= \int \sin x - \cos^2 x \sin x \, dx$

$= \int \sin x \, dx - \int \cos^2 x \sin x \, dx$

Let $u = \cos x$

$\frac{du}{dx} = -\sin x$

$du = -\sin x \, dx$

$= -\cos x + \int u^2 \, du$

$= -\cos x + \frac{u^3}{3} + c$

$= -\cos x + \frac{\cos^3 x}{3} + c$

Q6. $\int \cos^3 x \, dx$

$= \int (1 - \sin^2 x) \cos x \, dx$

$= \int \cos x \, dx - \int \sin^2 x \cos x \, dx$

Let $u = \sin x$

$\frac{du}{dx} = \cos x$

$du = \cos x \, dx$

$= \sin x - \int u^2 \, du$

$= \sin x - \frac{u^3}{3} + c$

$= \sin x - \frac{\sin^3 x}{3} + c$

Q7. $\int \cos^5 x \, dx$

$= \int \cos^4 x \cos x \, dx$

$= \int (1 - \sin^2 x)^2 \cos x \, dx$

$= \int (1 - 2\sin^2 x + \sin^4 x) \cos x \, dx$

Let $u = \sin x$

$\frac{du}{dx} = \cos x$

$du = \cos x \, dx$

$$= \int 1 - 2u^2 + u^4 \, du$$
$$= u - \frac{2u^3}{3} + \frac{u^5}{5} + c$$
$$= \sin x - \frac{2\sin^3 x}{3} + \frac{\sin^5 x}{5} + c$$

Q8 $\int \cos^2 x \, dx$

$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= \cos^2 x - 1 + \cos^2 x$$
$$= 2\cos^2 x - 1$$
$$\frac{1}{2} + \frac{1}{2}\cos 2x = \cos^2 x$$
$$\therefore \int \frac{1}{2} + \frac{1}{2}\cos 2x \, dx$$
$$= \frac{1}{2}x + \frac{1}{4}\sin 2x + c$$

Q9. $\int \sin^2 x \, dx$.

$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 1 - \sin^2 x - \sin^2 x$$
$$= 1 - 2\sin^2 x$$
$$\therefore \frac{1}{2} - \frac{\cos 2x}{2} = \sin^2 x$$
$$\int \frac{1}{2} - \frac{1}{2}\cos 2x \, dx$$
$$= \frac{1}{2}x - \frac{1}{4}\sin 2x + c$$

Q10. $\int 8\sin^4 x \, dx$

$$= \int 8\left(\frac{1}{2} - \frac{1}{2}\cos 2x\right)^2 dx$$
$$= \int 8\left(\frac{1}{4} - \frac{1}{2}\cos 2x + \frac{1}{4}\cos^2 2x\right) dx$$
$$= \int 2 - 4\cos 2x + 2\cos^2 2x \, dx$$
$$= \int 2 - 4\cos 2x + (1 + \cos 4x) \, dx$$
$$= \int 3 - 4\cos 2x + \cos 4x \, dx$$
$$= 3x - 2\sin 2x + \frac{1}{4}\sin 4x + c$$

Q11. $\int \cos^2 x + \sin^2 x \, dx$

$$= \int \frac{1}{2} + \frac{1}{2}\cos 2x + \frac{1}{2} - \frac{1}{2}\cos 2x \, dx$$

or :

$$= \int 1 \, dx$$
$$= x + c$$

Q12 $\int \cos^2 x - \sin^2 x \, dx$

$$= \int \cos 2x \, dx$$
$$= \frac{1}{2}\sin 2x + c$$

Q13 $\int \sin^3 x + \cos^2 x \, dx$

$$= \int (1 - \cos^2 x)\sin x + \cos^2 x \, dx$$
$$= \int \sin x - \cos^2 x \sin x + \frac{1}{2} + \frac{1}{2}\cos 2x \, dx$$
$$= -\cos x + \frac{\cos^3 x}{3} + \frac{1}{2}x + \frac{1}{4}\cos 2x +$$

Q14. $\int 2\sin x \cos x \, dx$

$$= \int \sin 2x \, dx$$
$$= -\frac{1}{2}\cos 2x + c$$

Q15. $\int \sin^3 x \cos^2 x \, dx$

$$= \int \sin^3 x (1 - \sin^2 x) \, dx$$
$$= \int \sin^3 x - \sin^5 x \, dx$$
$$= \int \sin^3 x \, dx - \int \sin^5 x \, dx$$
$$= \int \sin x - \cos^2 x \sin x \, dx - \int (1 - \cos^2 x)^2 \sin x \, dx$$
$$= -\cos x + \frac{\cos^3 x}{3} - \int (1 - 2\cos^2 x + \cos^4 x)\sin x \, dx$$
$$= -\cos x + \frac{\cos^3 x}{3} - \int \sin x - 2\cos^2 x \sin x + \cos^4 x \sin x \, dx$$
$$= -\cos x + \frac{\cos^3 x}{3} + \cos x - \frac{2}{3}\cos^3 x + \frac{1}{5}\cos^5 x + c$$
$$= -\frac{1}{3}\cos^3 x + \frac{1}{5}\cos^5 x + c$$

Q16. $\int \cos^3x \sin^2x \, dx$

$= \int \cos^3x(1-\cos^2x)\, dx$

$= \int \cos^3x - \cos^5x \, dx$

$= \int \cos^3x \, dx - \int \cos^5x \, dx$

$= \sin x - \frac{\sin^3x}{3} - \left[\sin x - \frac{2}{3}\sin^3x + \frac{1}{5}\sin^5x\right] + c$

$= \frac{1}{3}\sin^3x - \frac{1}{5}\sin^5x + c$

Q17. $\int \tan^2 3x \, dx$

$\tan^2\theta + 1 = \sec^2\theta$

$= \int \sec^2 3x - 1 \, dx$

$= \frac{1}{3}\tan 3x - x + c$

Q18. $\int 1 + \tan^2 x \, dx$

$= \int \sec^2x \, dx$

$= \tan x + c$

Q19. $\int \frac{\sin x}{1-\sin x} \times \frac{\sin x}{1+\sin x} \, dx$

$= \int \frac{\sin^2x}{1-\sin^2x} \, dx$

$= \int \frac{\sin^2x}{\cos^2x} \, dx$

$= \int \tan^2 x \, dx$

$= \int \sec^2x - 1 \, dx$

$= \tan x - x + c$

Q20. $\int \sec^2x \tan^4 x \, dx$

Let $u = \tan x$

$\frac{du}{dx} = \sec^2x$

$du = \sec^2x \, dx$

$= \int u^4 \, du$

$= \frac{u^5}{5} + c$

$= \frac{\tan^5x}{5} + c$

Q21. $A = \int_0^{2\pi} x + \cos^2x - \sin^2x \, dx$

$= \int_0^{2\pi} x + \cos 2x \, dx$

$= \left[\frac{x^2}{2} + \frac{1}{2}\sin 2x\right]_0^{2\pi}$

$= \frac{(2\pi)^2}{2} + 0 - 0 + 0$

$= \frac{4\pi^2}{2}$

$= 2\pi^2$ units

Q22. $\underset{\sim}{v}(t) = 4\sin^2t\,\underset{\sim}{i} + \tan^2t\,\underset{\sim}{j}$

a) $\underset{\sim}{r}(t) = \int 4\sin^2t\,\underset{\sim}{i} + \tan^2t\,\underset{\sim}{j} \, dt$

$= \int \begin{pmatrix} 2-2\cos 2t \\ \sec^2t - 1 \end{pmatrix} dt$

$= \begin{pmatrix} 2t - \sin 2t \\ \tan t - t \end{pmatrix} + \underset{\sim}{c}$

$\begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \underset{\sim}{c}$

$\therefore \underset{\sim}{r}(t) = (3+2t-\sin 2t)\underset{\sim}{i} + (1+\tan t - t)\underset{\sim}{j}$

b) $\underset{\sim}{r}\left(\frac{\pi}{4}\right) = \left(2+\frac{\pi}{2}\right)\underset{\sim}{i} + \left(2-\frac{\pi}{4}\right)\underset{\sim}{j}$

EXERCISE 9E

Q1. $\int \frac{7}{x}\,dx$

$= 7\ln|x| + c.$

Q2. $\int 3x^2 - \frac{4}{x}\,dx$

$= x^3 - 4\ln|x| + c$

Q3. $\int \frac{8x}{x^2+6}\,dx$

$\frac{d}{dx}(x^2+6) = 2x$

$\therefore = 4\ln(x^2+6) + c$

Q4. $\int \tan 2x\,dx$

$= \int \frac{\sin 2x}{\cos 2x}\,dx$

$\frac{d}{dx}(\cos 2x) = -2\sin(2x)$

$= -\frac{1}{2}\ln|\cos 2x| + c.$

Q5. $\int \frac{x+2}{x}\,dx$

$= \int 1 + \frac{2}{x}\,dx$

$= x + 2\ln|x| + c$

Q6. $\int \frac{x}{x+2}\,dx.$

$\frac{x}{x+2} = 1 - \frac{2}{x+2}$

$\therefore \int 1 - \frac{2}{x+2}\,dx$

$= x - 2\ln|x+2| + c$

Q7. $\int \frac{2x-3}{x}\,dx$

$= \int 2 - \frac{3}{x}\,dx$

$= 2x - 3\ln|x| + c$

Q8 $\int \frac{x}{2x-3}\,dx$

$$\begin{array}{r|l} & \frac{1}{2} \\ 2x-3 & x \\ & -(x-\frac{3}{2}) \\ \hline & \frac{3}{2}. \end{array}$$

$\therefore \int \frac{1}{2} + \frac{3}{2(2x-3)}\,dx$

$= \frac{1}{2}x + \frac{3}{4}\ln|2x-3| + c$

Q9. $\int \frac{x^2+4x+1}{x+3}\,dx$

$$\begin{array}{r|rrr} -3 & 1 & 4 & 1 \\ & & -3 & -3 \\ \hline & 1 & 1 & -2 \end{array}$$

$= \int x + 1 - \frac{2}{x+3}\,dx$

$= \frac{x^2}{2} + x - 2\ln|x+3| + c$

Q10. $\int \frac{5x+3}{x(x+1)}\,dx$

$\frac{A}{x} + \frac{B}{x+1} = \frac{5x+3}{x(x+1)}$

$Ax + A + Bx = 5x + 3.$

$A + B = 5$

$A = 3$

$\therefore B = 2$

$\therefore \int \frac{3}{x} + \frac{2}{x+1}\,dx$

$= 3\ln|x| + 2\ln|x+1| + c$

Q11. $\displaystyle\int \frac{4x-7}{(x+2)(x-3)}\,dx$

$$\frac{A}{x+2} + \frac{B}{x-3} = \frac{4x-7}{(x+2)(x-3)}$$

$Ax - 3A + Bx + 2B = 4x - 7$

$A + B = 4 \Rightarrow B = 4 - A$

$2B - 3A = -7$

$2(4-A) - 3A = -7$

$8 - 2A - 3A = -7$

$-5A = -15$

$A = 3$

$\therefore B = 1$

$$\therefore \int \frac{3}{x+2} + \frac{1}{x-3}\,dx$$

$= 3\ln|x+2| + \ln|x-3| + c$

Q.2. $\displaystyle\int \frac{5x^2-2x+18}{(x-1)(x^2+6)}\,dx$

$$\frac{A}{x-1} + \frac{Bx+C}{x^2+6} = \frac{5x^2-2x+18}{(x-1)(x^2+6)}$$

$Ax^2 + 6A + Bx^2 + Cx - Bx - C = 5x^2 - 2x + 18$

$A + B = 5$

$C - B = -2 \Rightarrow C = B - 2$

$6A - C = 18$

$6A - B + 2 = 18$

$6A - B = 16$

$A + B = 5$

$7A = 21$

$A = 3$

$\therefore B = 2$ and $C = 0$

$$\int \frac{3}{x-1} + \frac{2x}{x^2+6}\,dx$$

$= 3\ln|x-1| + \ln(x^2+6) + c$

Q13. $\displaystyle\int \frac{7x^2+8x-4}{(x+1)(x^2+x-1)}\,dx$

$$\frac{A}{x+1} + \frac{Bx+C}{x^2+x-1} = \frac{7x^2+8x-4}{(x+1)(x^2+x-1)}$$

$Ax^2 + Ax - A + Bx^2 + Cx + Bx + C = 7x^2 + 8x - 4$

$A + B = 7$

$A + B + C = 8 \Rightarrow 7 + C = 8$

$-A + C = -4$ $\quad C = 1$

$-A + 1 = -4$

$-A = -5$

$A = 5$

$\therefore B = 2$

$$\therefore \int \frac{5}{x+1} + \frac{2x+1}{x^2+x-1}\,dx$$

$= 5\ln|x+1| + \ln|x^2+x-1| + c$

Q14. $\displaystyle\int \frac{5x^2-10x-3}{(x+1)(x-1)^2}\,dx$

$$\frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{(x-1)^2} = \frac{5x^2-10x-3}{(x+1)(x-1)^2}$$

When $x = -1$, $\dfrac{5(1)+10-3}{4}$

$A = 3$

When $x = 1$, $\dfrac{5-10-3}{2}$

$C = -4$

when $x = 0$, $\dfrac{3}{1} + \dfrac{B}{-1} - \dfrac{4}{1} = \dfrac{-3}{1}$

$-B - 1 = -3$

$B = 2$

$$= \int \frac{3}{x+1} + \frac{2}{x-1} - \frac{4}{(x-1)^2}\, dx$$

$$= 3\ln|x+1| + 2\ln|x-1| - \int \frac{4}{(x-1)^2}\, dx$$

Let $u = x-1$

$$\frac{du}{dx} = 1$$

$$dx = du$$

$$= 3\ln|x+1| + 2\ln|x-1| - \int 4u^{-2}\, du.$$

$$= 3\ln|x+1| + 2\ln|x-1| - \frac{4u^{-1}}{-1} + c$$

$$= 3\ln|x+1| + 2\ln|x-1| + \frac{4}{x-1} + c$$

Q15. $\int \frac{8x^2 - 44x + 25}{(2x+1)(x-3)^2}\, dx$

$$\frac{A}{2x+1} + \frac{B}{x-3} + \frac{C}{(x-3)^2} = \frac{8x^2-44x+25}{(2x+1)(x-3)^2}$$

When $x = -\frac{1}{2}$, $\frac{8(\frac{1}{4}) + 22 + 25}{(\frac{-7}{2})^2}$

$$A = \frac{2+47}{(\frac{49}{4})}$$

$$= 49 \div \frac{49}{4}$$

$$A = 4$$

When $x = 3$, $\frac{72 - 132 + 25}{7}$

$$C = \frac{-35}{7}$$

$$C = -5$$

When $x = 0$, $\frac{4}{1} + \frac{B}{-3} + \frac{-5}{9} = \frac{25}{9}$

$$36 - 3B - 5 = 25$$

$$31 - 3B = 25$$

$$-3B = -6$$

$$B = 2$$

$$\therefore \int \frac{4}{2x+1} + \frac{2}{x-3} - \frac{5}{(x-3)^2}\, dx$$

$$= 2\ln|2x+1| + 2\ln|x-3| + \frac{5}{x-3} + c.$$

Q16. Intersection points.

$$\frac{x}{x-2} = \frac{11x}{x^2+2}$$

$$x^3 + 2x = 11x^2 - 22x$$

$$x^3 - 11x^2 + 24x = 0$$

$$x(x^2 - 11x + 24) = 0$$

$$x(x-3)(x-8) = 0$$

$x = 0$, $x = 3$, $x = 8$

$$\therefore \int_3^8 \frac{11x}{x^2+2} - \frac{x}{x-2}\, dx$$

$$= \int_3^8 \frac{11x}{x^2+2} - \left(1 + \frac{2}{x-2}\right) dx$$

$$= \left[\frac{11}{2}\ln(x^2+2) - x - 2\ln|x-2|\right]_3^8$$

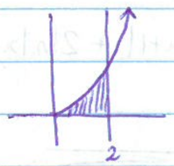
$$= \frac{11}{2}\ln(66) - 8 - 2\ln(6)$$
$$- \frac{11}{2}\ln(11) + 3 + 2\ln(1)$$

$$= \frac{11}{2}\ln(6) - 2\ln(6) - 5$$

$$= \frac{7}{2}\ln(6) - 5 \text{ units}^2$$

EXERCISE 9F

Q1.



$$V = \pi \int_0^2 (x^2)^2 \, dx$$

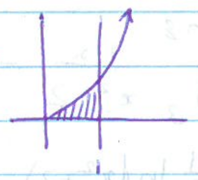
$$= \pi \int_0^2 x^4 \, dx$$

$$= \pi \left[\frac{x^5}{5} \right]_0^2$$

$$= \pi \left(\frac{32}{5} \right)$$

$$= \frac{32\pi}{5} \text{ units}^3$$

Q2.



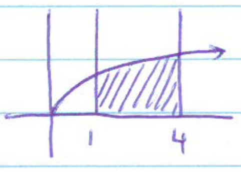
$$V = \pi \int_0^1 (3x^2)^2 \, dx$$

$$= \pi \int_0^1 9x^4 \, dx$$

$$= \pi \left[\frac{9x^5}{5} \right]_0^1$$

$$= \frac{9\pi}{5} \text{ units}^3$$

Q3.



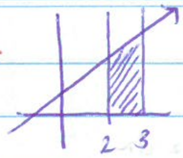
$$V = \pi \int_1^4 (\sqrt{x})^2 \, dx$$

$$= \pi \left[\frac{x^2}{2} \right]_1^4$$

$$= \pi \left(8 - \frac{1}{2} \right)$$

$$= \frac{15\pi}{2} \text{ units}^3$$

Q4.



$$V = \pi \int_2^3 (2x+1)^2 \, dx$$

$$= \pi \int_2^3 4x^2 + 4x + 1 \, dx$$

$$= \pi \left[\frac{4x^3}{3} + \frac{4x^2}{2} + x \right]_2^3$$

$$= \pi \left[36 + 18 + 3 - \left(\frac{4}{3}(8) + 8 + 2 \right) \right]$$

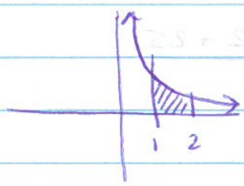
$$= \pi \left[57 - \frac{56}{3} - \frac{6}{3} \right]$$

$$= \pi \left[57 - \frac{62}{3} \right]$$

$$= \pi \left[\frac{171}{3} - \frac{62}{3} \right]$$

$$= \frac{109\pi}{3} \text{ units}^3$$

Q5 a)



$$V = \pi \int_1^2 \left(\frac{1}{x} \right)^2 dx$$

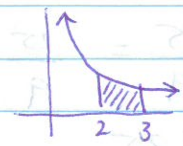
$$= \pi \int_1^2 \frac{1}{x^2} \, dx$$

$$= \pi \left[-\frac{1}{x} \right]_1^2$$

$$= \pi \left[-\frac{1}{2} - (-1) \right]$$

$$= \frac{\pi}{2} \text{ units}^3$$

b)



$$V = \pi \int_2^3 \frac{1}{x^2} \, dx$$

$$= \pi \left[-\frac{1}{x} \right]_2^3$$

$$= \pi \left[-\frac{1}{3} + \frac{1}{2} \right]$$

$$= \pi \left[\frac{3-2}{6} \right]$$

$$= \frac{\pi}{6} \text{ units}^3$$

Q6.

$$V = \pi \int_{-1}^{2} (x^2+1)^2 \, dx$$

$$= \pi \int_{-1}^{2} x^4 + 2x^2 + 1 \, dx$$

$$= \pi \left[\frac{x^5}{5} + \frac{2x^3}{3} + x \right]_{-1}^{2}$$

$$= \pi \left[\frac{32}{5} + \frac{16}{3} + 2 - \left(-\frac{1}{5} - \frac{2}{3} - 1 \right) \right]$$

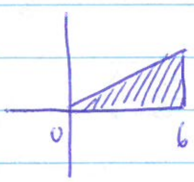
$$= \pi \left(\frac{96}{15} + \frac{80}{15} + \frac{30}{15} - \left(-\frac{3}{15} - \frac{10}{15} - \frac{15}{15} \right) \right)$$

$$= \pi \left(\frac{206}{15} + \frac{28}{15} \right)$$

$$= \frac{234\pi}{15}$$

$$= \frac{78\pi}{5} \text{ units}^3$$

Q7.



$$V = \pi \int_{0}^{6} (0.5x)^2 \, dx$$

$$= \pi \int_{0}^{6} \frac{1}{4} x^2 \, dx$$

$$= \pi \left[\frac{x^3}{12} \right]_{0}^{6}$$

$$= \pi \left(\frac{216}{12} - 0 \right)$$

$$= \frac{216\pi}{12} \text{ units}^3$$

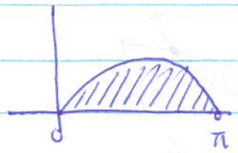
$$= 18\pi \text{ units}^3$$

Check: $V = \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\pi (3)^2 (6)$$

$$= 18\pi \text{ units}^3$$

Q8.



$$V = \pi \int_{0}^{\pi} (\sqrt{\sin x})^2 \, dx$$

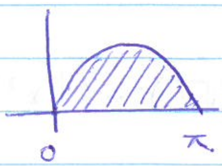
$$= \pi \int_{0}^{\pi} \sin x \, dx$$

$$= \pi \left[-\cos x \right]_{0}^{\pi}$$

$$= \pi (1 - (-1))$$

$$= 2\pi \text{ units}^3$$

Q9.



$$V = \pi \int_{0}^{\pi} \sin^2 x \, dx$$

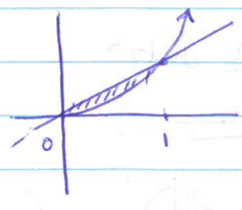
$$= \pi \int_{0}^{\pi} \frac{1}{2} - \frac{1}{2} \cos 2x \, dx$$

$$= \pi \left[\frac{1}{2}x - \frac{1}{4} \sin 2x \right]_{0}^{\pi}$$

$$= \pi \left(\frac{1}{2}\pi - 0 - 0 \right)$$

$$= \frac{\pi^2}{2} \text{ units}^3$$

Q10.



(washer method).

$$V = \pi \int_{0}^{1} x^2 - (x^2)^2 \, dx.$$

$$= \pi \int_{0}^{1} x^2 - x^4 \, dx$$

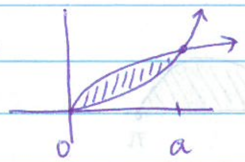
$$= \pi \left[\frac{x^3}{3} - \frac{x^5}{5} \right]_{0}^{1}$$

$$= \pi \left[\frac{1}{3} - \frac{1}{5} \right]$$

$$= \pi \left[\frac{5-3}{15} \right]$$

$$= \frac{2\pi}{15} \text{ units}^3$$

Q11.



$$\frac{1}{8}x^2 = \sqrt{x}$$

$$\frac{1}{64}x^4 = x$$

$$\frac{1}{64}x^4 - x = 0$$

$$\frac{1}{64}x(x^3 - 64) = 0$$

$$\underline{\underline{x = 0}} \quad \text{or} \quad \underline{\underline{x = 4}}$$

$$\therefore a = 4$$

$$V = \pi \int_0^4 (\sqrt{x})^2 - \left(\tfrac{1}{8}x^2\right)^2 \, dx$$

$$= \pi \int_0^4 x - \frac{1}{64}x^4 \, dx$$

$$= \pi \left[\frac{x^2}{2} - \frac{x^5}{320} \right]_0^4$$

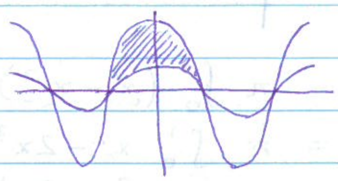
$$= \pi \left[8 - \frac{1024}{320} \right]$$

$$= \pi \left(\frac{2560 - 1024}{320} \right)$$

$$= \frac{1536}{320}\pi$$

$$= \underline{\underline{\frac{24\pi}{5} \text{ units}^3}}$$

Q12.



$$V = \pi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (3\cos x)^2 - (\cos x)^2 \, dx$$

$$= \pi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 9\cos^2 x - \cos^2 x \, dx$$

$$= \pi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 8\cos^2 x \, dx$$

$$= \pi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 8\left(\tfrac{1}{2} + \tfrac{1}{2}\cos 2x\right) dx$$

$$= \pi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 4 + 4\cos 2x \, dx$$

$$= \pi \left[4x + 2\sin 2x \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= \pi \left(2\pi + 0 + 2\pi + 0 \right)$$

$$= \underline{\underline{4\pi^2 \text{ units}^3}}$$

Q13.

$$V = \pi \int_{-r}^{r} \left(\sqrt{r^2 - x^2}\right)^2 dx$$

$$= \pi \int_{-r}^{r} r^2 - x^2 \, dx$$

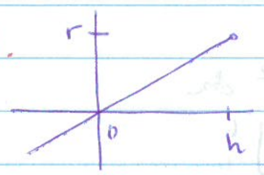
$$= \pi \left[r^2 x - \frac{x^3}{3} \right]_{-r}^{r}$$

$$= \pi \left(r^3 - \frac{r^3}{3} + r^3 - \frac{r^3}{3} \right)$$

$$= \pi \left(2r^3 - \frac{2r^3}{3} \right)$$

$$= \underline{\underline{\frac{4\pi r^3}{3}}}$$

Q14.



$$y = \frac{r}{h}x$$

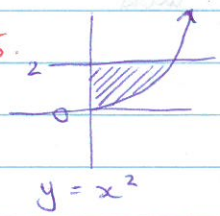
$$V = \pi \int_0^h \frac{r^2}{h^2} x^2 \, dx$$

$$= \pi \left[\frac{r^2}{h^2} \frac{x^3}{3} \right]_0^h$$

$$= \pi \left(\frac{r^2 h}{3} \right)$$

$$= \underline{\underline{\tfrac{1}{3}\pi r^2 h}}$$

Q15.

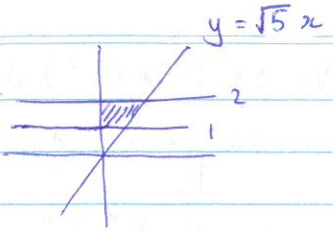


$$V = \pi \int_0^2 y \, dy$$

$$= \pi \left[\frac{y^2}{2} \right]_0^2$$

$$= \underline{\underline{2\pi \text{ units}^3}}$$

Q16.



$$V = \pi \int_1^2 \left(\frac{y}{\sqrt{5}}\right)^2 dy$$

$$= \pi \int_1^2 \frac{y^2}{5}\, dy$$

$$= \pi \left[\frac{y^3}{15}\right]_1^2$$

$$= \pi\left(\frac{8}{15} - \frac{1}{15}\right)$$

$$= \frac{7\pi}{15} \text{ units}^3$$

Check.

$$V_L = \frac{1}{3}\pi\left(\frac{2}{\sqrt{5}}\right)^2(2)$$

$$= \frac{2}{3}\pi\left(\frac{4}{5}\right)$$

$$= \frac{8}{15}\pi$$

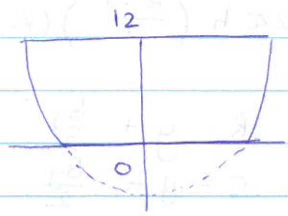
$$V_S = \frac{1}{3}\pi\left(\frac{1}{\sqrt{5}}\right)^2(1)$$

$$= \frac{1}{3}\pi\left(\frac{1}{5}\right)$$

$$= \frac{1}{15}\pi$$

$$\therefore \left(\frac{8}{15} - \frac{1}{15}\right)\pi = \frac{7}{15}\pi$$

Q17.



$y = x^2 - 3$

$y + 3 = x^2$

$$V = \pi \int_0^{12} y + 3\, dy$$

$$= \pi\left[\frac{y^2}{2} + 3y\right]_0^{12}$$

$$= \pi\left[\frac{144}{2} + 36\right]$$

$$= 108\pi \text{ cm}^3.$$

Q18

$$V = \pi \int_1^4 (\sqrt{x})^2 - (\sqrt{x-1})^2\, dx + \pi \int_0^1 x\, dx$$

$$= \pi \int_1^4 x - x + 1\, dx + \pi \int_0^1 x\, dx$$

$$= \pi \int_1^4 1\, dx + \pi \int_0^1 x\, dx$$

$$= \pi\left[x\right]_1^4 + \pi\left[\frac{x^2}{2}\right]_0^1$$

$$= 4\pi - \pi + \frac{\pi}{2}$$

$$= \frac{7\pi}{2} \text{ units}^3$$

Q19.

OPTION 1: $V = \pi \int_0^{\frac{\pi}{2}} \left(\frac{\sin x}{2}\right)^2 dx$

$$= \pi \int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{4}\, dx$$

$$= \frac{\pi}{4} \int_0^{\frac{\pi}{2}} \frac{1}{2} - \frac{1}{2}\cos 2x\, dx$$

$$= \frac{\pi}{4}\left[\frac{1}{2}x - \frac{1}{4}\sin 2x\right]_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{4}\left[\frac{\pi}{4}\right]$$

$$= \frac{\pi^2}{16} \text{ m}^3$$

OPTION 2: $V = \pi \int_0^{\frac{\pi}{2}} \frac{x}{2\pi}\, dx$

$$= \frac{1}{2} \int_0^{\frac{\pi}{2}} x\, dx$$

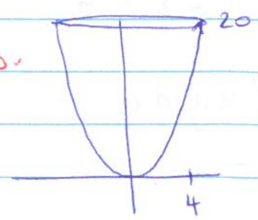
$$= \frac{1}{2}\left[\frac{x^2}{2}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{2}\left(\frac{\pi^2}{8}\right)$$

$$= \frac{\pi^2}{16} \text{ m}^3$$

$\therefore$ Both the same volume.

Q20.



$y = kx^2$

$20 = k(16)$

$k = \frac{5}{4}$

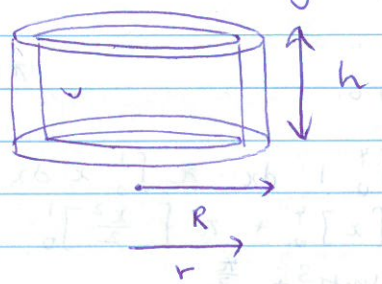
$x^2 = \frac{4}{5}y$

$$\therefore V = \pi \int_0^{20} \frac{4}{5}y\, dy$$

$$= \pi\left[\frac{4y^2}{10}\right]_0^{20}$$

$$= 160\pi \text{ units}^3$$

Q21. Consider a cylindrical shell



$$V = \pi R^2 h - \pi r^2 h$$
$$= \pi h(R^2 - r^2)$$
$$= \pi h(R+r)(R-r)$$
$$= 2\pi h\left(\frac{R+r}{2}\right)(R-r)$$

2π height average radius thickness.

Putting this into the context of a function $y = f(x)$, with shell thickness δx, height y.

A shell thickness of δx is obtained by assuming

$$R = x + \frac{\delta x}{2} \text{ and}$$
$$r = x - \frac{\delta x}{2}$$

as $x + \frac{\delta x}{2} - x + \frac{\delta x}{2}$

$= \delta x$

$$\therefore V = 2\pi y\left(\frac{2x}{2}\right)\delta x$$

$$V = 2\pi xy\,\delta x$$

For an interval $a \le x \le b$,

$$\therefore V = 2\pi \int_a^b xy\,dx$$

(a) $V = 2\pi \int_1^2 x(x^2)\,dx$

$= 2\pi \int_1^2 x^3\,dx$

$= 2\pi \left[\frac{x^4}{4}\right]_1^2$

$= 2\pi\left(4 - \frac{1}{4}\right)$

$= \frac{30\pi}{4}$ units^3 $\left(\frac{15\pi}{2} \text{ units}^3\right)$.

b) $V = 2\pi \int_1^4 x(1+\sqrt{x})\,dx$

$= 2\pi \int_1^4 x + x^{\frac{3}{2}}\,dx$

$= 2\pi \left[\frac{x^2}{2} + \frac{2x^{\frac{5}{2}}}{5}\right]_1^4$

$= 2\pi\left[8 + \frac{64}{5} - \frac{1}{2} - \frac{2}{5}\right]$

$= 2\pi\left(\frac{80}{10} + \frac{128}{10} - \frac{5}{10} - \frac{4}{10}\right)$

$= 2\pi\left(\frac{208-9}{10}\right)$

$= \frac{199\pi}{5}$ units^3

Q22. By similar principle.

$$V = 2\pi h\left(\frac{R+r}{2}\right)(R-r)$$

where $R = y + \frac{\delta y}{2}$

$r = y - \frac{\delta y}{2}$

$\therefore V = 2\pi xy\,\delta y$.

$V = 2\pi \int_a^b xy\,\delta y$

a) $V = 2\pi \int_1^2 x\left(\frac{1}{x}\right)\delta y$

$= 2\pi \left[y\right]_1^2$

$= 2\pi(2-1) = 2\pi$ units^3

b) $V = 2\pi \int_1^2 y\left(\frac{y}{2}\right)dy$

$= 2\pi\left[\frac{y^3}{6}\right]_1^2$

$= 2\pi\left(\frac{8}{6} - \frac{1}{6}\right)$

$= \frac{14\pi}{6} = \frac{7\pi}{3}$ units^3